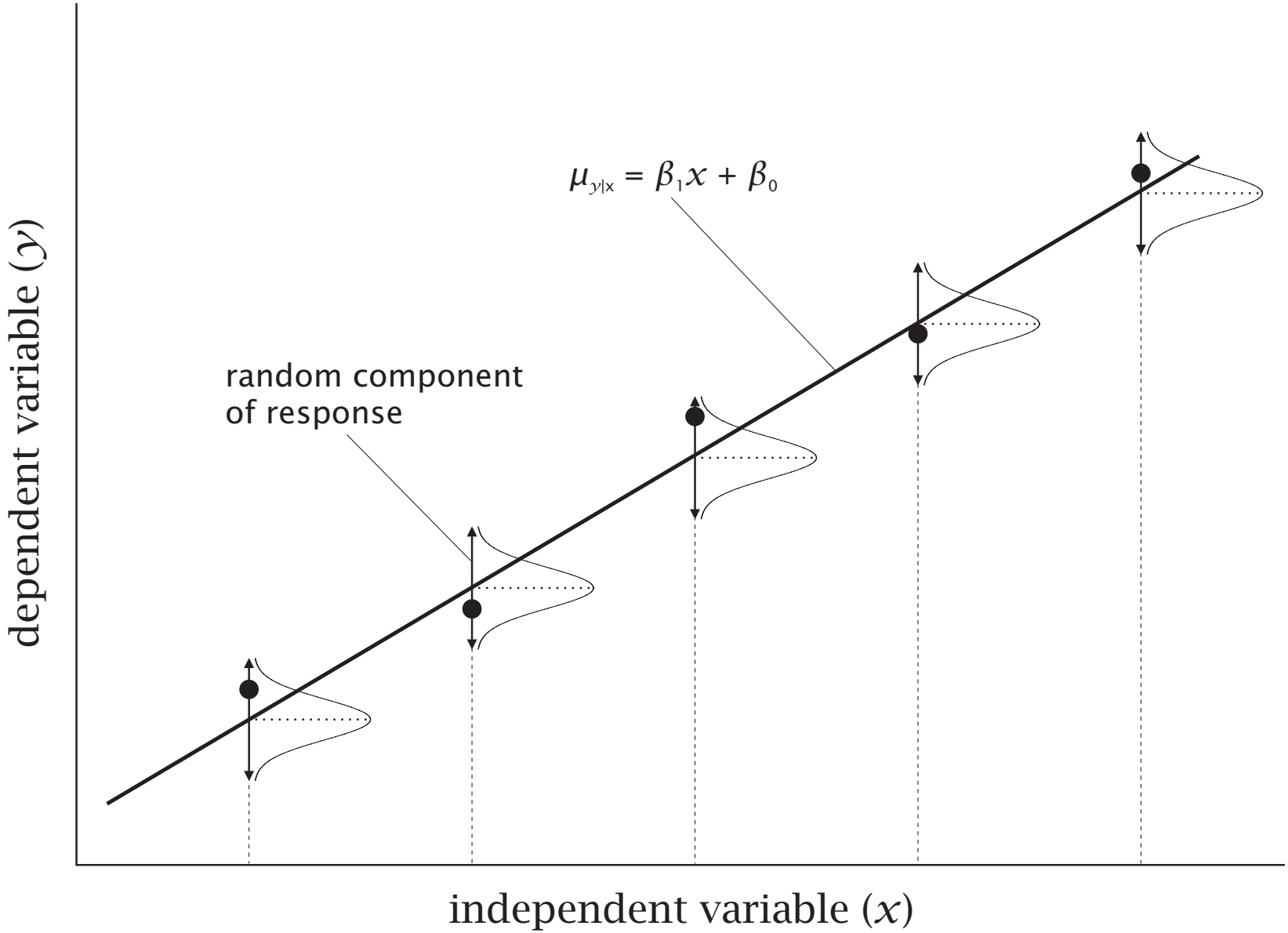
## Regression Model

Can you remind me of the framework of regression analysis?



## Prediction in Regression Analysis

What are the types of predictions usually needed with regression analysis?

- Confidence intervals for $\mu_{y|x}$ at a given value of $x$.
- Prediction intervals for future values of $y_u$ at a given value of $x_u$
- Inverse regression: confidence intervals for $x_u$ (concentration unknown) that generated a measured value $y_u$

## Confidence Band

What is the confidence band of a fitted regression model?

- The standard error of the fitted value $\hat{y}_u$ at $x = x_u$ can be estimated for a best-fit line obtained by least-squares regression:

$$s(\hat{y}_u) = s_{\text{res}} \sqrt{\frac{1}{n} + \frac{(x_u - \bar{x})^2}{S_{xx}}} \qquad \text{(OLS of SLR model)}$$

$$= \frac{s_{\text{res}}}{\sqrt{n}} + s_{\text{res}} \sqrt{\frac{(x_u - \bar{x})^2}{S_{xx}}} \qquad \text{where } S_{xx} = \sum (x_i - \bar{x})^2$$

- A parametric confidence interval can be calculated from $\hat{y} \pm t_{\text{crit}} \cdot s(\hat{y})$, which assumes that $\hat{y}$ is normally distributed.
- A *confidence band* is a plot of the CI of $\mu_y$ as a function of $x$

## Prediction Band

What is the prediction band of a fitted regression model?

- A prediction interval contains future conditional measurements with the stated level of confidence
- For OLS regression at $x = x_u$, the std error of the residuals for observation $y_u$ can be estimated by

$$s(e_u) = s_{\text{res}} \sqrt{1 + \frac{1}{n} + \frac{(x_u - \bar{x})^2}{S_{xx}}} \qquad \text{(OLS of SLR model)}$$

  More generally,

$$\sigma^2(e_u) = \sigma^2 + \sigma^2(\hat{y}_u) \qquad \text{(homogeneous variance)}$$

- A prediction interval can be calculated as $\hat{y} \pm t_{\text{crit}} \cdot s(e_u)$, which assumes the errors on the $y$ measurements are normally distributed
- A *prediction band* is a plot of the PI as a function of $x$. It is wider than the confidence band.