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- Let f be a function of several random variables x_i. The variance of the function is

$$\sigma_f^2 = \sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2 + \sum_i \sum_{j(j\neq i)} \left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\sigma_{ij}$$

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- σ_{ij} is the covariance of variables x_i and x_j, while ρ_{ij} is the correlation coefficient between the variables.

$$\sigma_{ij} = E[(x_i - \mu_x)(y_i - \mu_y)]$$
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- More generally, to propagate uncertainty intervals Δx_i (eg, MOEs or MADs):

$$(\Delta f)^2 = \sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 (\Delta x_i)^2 + \sum_i \sum_{j(j\neq i)} \left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\Delta x_i \Delta x_j \rho_{ij}$$

Propagation of Correlated Errors

Apply the error propagation equation to the calculation of the flow rate u_{opt} that optimizes separation according to the van Deemter equation.

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$$u_{\text{opt}} = \sqrt{\frac{B}{C}}$$

so with only two random variables the equation becomes

$$\sigma^2_{u_{\text{opt}}} = \left(\frac{\partial u_{\text{opt}}}{\partial B}\right)^2 \sigma^2_B + \left(\frac{\partial u_{\text{opt}}}{\partial C}\right)^2 \sigma^2_C + 2\left(\frac{\partial u_{\text{opt}}}{\partial B}\right)\left(\frac{\partial u_{\text{opt}}}{\partial C}\right) \rho_{BC}\sigma_B\sigma_C$$

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The partial derivatives in this case are

$$\frac{\partial u_{\text{opt}}}{\partial B} = \sqrt{\frac{1}{4BC}}$$

$$\frac{\partial u_{\text{opt}}}{\partial C} = -\sqrt{\frac{B}{4C^3}}$$

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Function	**Uncertainty of Result**
$y = x_1 + x_2$	$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2$
$y = x_1 - x_2$	$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2$
$y = x_1 x_2$	$\mathrm{RSD}_y^2 \approx \mathrm{RSD}_1^2 + \mathrm{RSD}_2^2 + 2\rho_{12}\mathrm{RSD}_1\mathrm{RSD}_2$
$y = \frac{x_1}{x_2}$	$\mathrm{RSD}_y^2 \approx \mathrm{RSD}_1^2 + \mathrm{RSD}_2^2 - 2\rho_{12}\mathrm{RSD}_1\mathrm{RSD}_2$